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LETTER TO THE EDITOR

Power-law distribution of landslides in an experiment on the erosion of a granular pile

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Abstract. Experimental aspects of self-organized criticality are investigated in a micro-model exhibiting mass displacements involving material flow in three dimensions. By watering a ridge-shaped pile made of a granular mixture we observe micro-landslides whose size distribution indicates a power-law dependence with an exponent τ close to 1. To obtain a more detailed description of the geometrical changes in the profile of the eroding ridge we also determine the temporal-scaling of the height correlation function and the distribution of the local velocities.

It has recently been postulated that many driven systems in nature are gradually approaching a self-organized critical (SOC) state [1] in which break-down phenomena of various origin result in a power-law distribution of avalanche-like transport of energy, mass, etc. The corresponding cellular-automata-type models [1–5] and the associated experiments [6–11] have been shown to display a rich behaviour; in some cases an algebraic decay of avalanches was detected with a non-trivial exponent.

The main objective of our work is to study a system which is relevant from the point of establishing connections among three important topics of great current interest: (i) self-organized criticality (SOC) [1]; (ii) kinetic roughening [12–14]; and (iii) the physics of granular materials [15]. In addition, our experiments on the erosion of a granular pile are expected to be illuminating in the context of geomorphological evolution as well.

The experimental set-up (see figure 1 and [16, 17]) consisted of a table of linear size 90 cm with channels attached to its edges. The granular pile made of a mixture of silica sand and pot soil (i.d. Floravin 1991/87131) was erected across the table and had the shape of a ridge. We sifted the sand and the soil and used mixtures with a well defined sand-to-soil ratio close to one (ranging from 0.8 to 1.2). Watering was carried out by implementing suitably modified commercial sprayers. During the experiments water was sprayed with an intensity 2000–3000 cc min^{-1} over the surface. As the water penetrated the granular pile, some of it became saturated by water and we could observe how these wet parts were sliding down along the surface or, in other cases, the local motion of the structure was more like a collapse than an avalanche. In the following we shall use the term 'landslide' for these relatively fast motions of masses of the experimental granular material.

Our purpose was to get information about the distribution of the sizes of landslides in our micromodel. A direct determination of the sliding masses hardly seemed feasible, thus, we applied the following procedure to obtain an estimate of the masses moving coherently close to the top part of the experimental structure.

(i) We took video recordings of the evolving pile and obtained the necessary information from the time dependence of the changes in the profile of the ridge.

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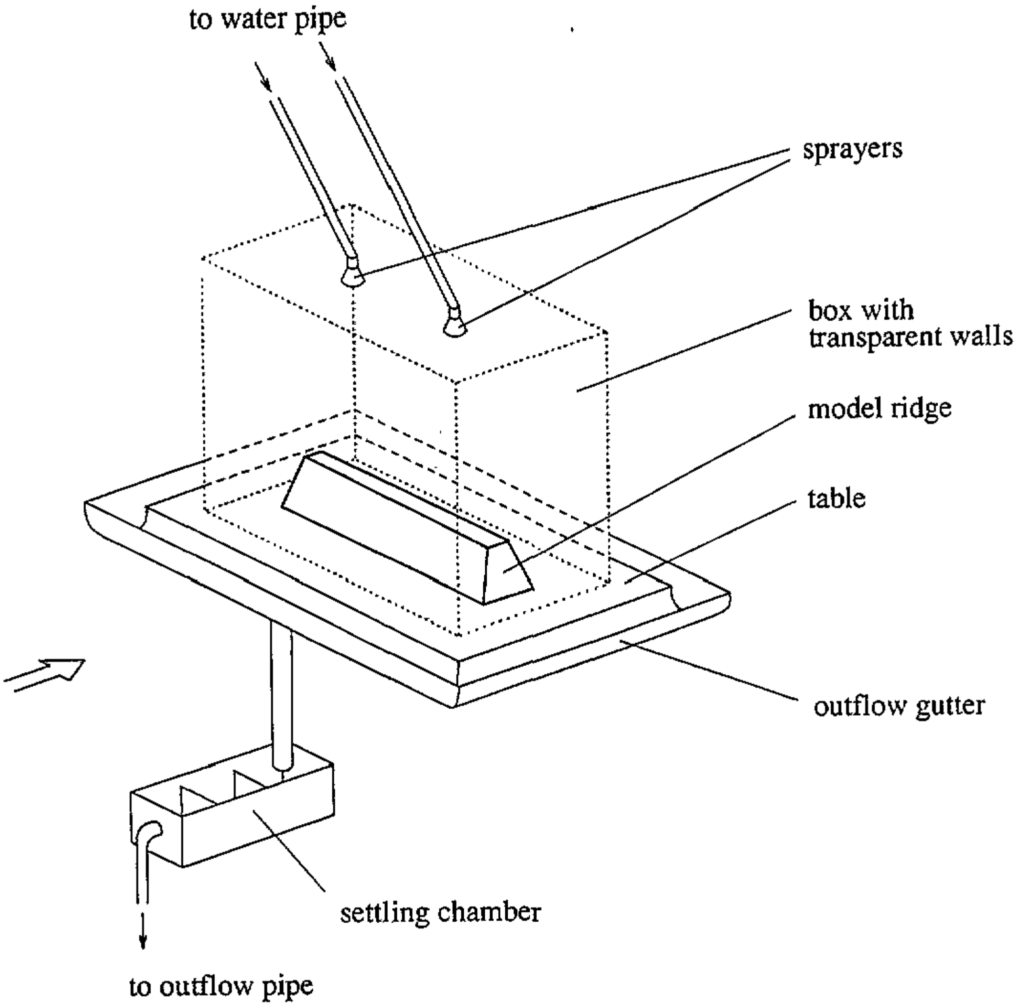


Figure 1. Sketch of the experimental set-up. The arrow on the left-hand side of the figure shows the position of the camera. White cardboard was illuminated on the opposite side of the table, providing white background behind the dark ridge in order to achieve sharp contrast and good resolution.

(ii) The actual size of the landslides (almost always originating at the top or 'profile' part of the pile) was estimated by taking into account that a landslide can be associated with a part of the surface which moves in a coherent manner with a velocity which is significantly larger than the surrounding parts of the structure. Consequently, the size s_i of the ith landslide was calculated from the expression

$$s_i = \int_{h_i} v(x, t)\, \mathrm{d}x\, \mathrm{d}t \qquad (1)$$

where v is the vertical component of the actual velocity of the surface (profile) at a given time t and horizontal position x, and h_i denotes the ith connected set of points in space-time for which v is beyond a critical value. In this way we obtained a quantity which could be associated with the projection of a landslide. Equation (1) expresses the fact that the size of a landslide was assumed to be proportional to its linear size, velocity and duration. Some of the approximations following from our assumptions will be discussed later in the paper.

(iii) The size of a landslide was calculated using a combination of several procedures during digital image processing of the data. At first we plotted the actual values of $v(x, t)$. In this space-time plot the magnitude of the velocity is displayed using a grey scale (darker shades correspond to larger velocities). The x coordinate is directed horizontally, while the vertical direction corresponds to time (as the sinking profile sweeps the rectangular area of this plot from top to bottom). A high-frequency filter was applied to eliminate the high-frequency digitization noise (figure 2(a)). Next we selected the points having larger velocity than the average velocity (≈ 3.5 mm s^{-1}) of the surface (figure 2(b)). To remove the effects of the non-zero average surface velocity, we used another appropriate digital filter (figure 2(c)). Finally the velocity space was smoothed (figure 2(d)). These type of pictures were evaluated using the Hoshen–Kopelman algorithm [18] providing the distribution of cluster sizes.

The above analysis was carried out for nine independent erosion experiments. There were about $n_{av} = 20$ landslides per experiment with n_{av} fluctuating between 10 and 30. To improve the statistics the results were accumulated into a single landslide size distribution which is shown in figure 3. The data demonstrate that $N(s)$, the cumulative number of landslides (clusters of size smaller than s), has an almost logarithmic dependence on s in a relatively wide region of sizes. This fact corresponds to a density distribution

$$p(s)\,\mathrm{d}s \sim s^{-\tau}\,\mathrm{d}s \tag{3}$$

with $\tau \simeq 1$, thus our data are consistent with the assumption of an algebraically decaying landslide size distribution (as one would expect in a self-organized critical system).

This result was obtained for the projection of the landslides. Interestingly, the conclusion is not changed if we assume that the volume V of a landslide is proportional to $V \sim s^{1/2}s$ (taking into account the three-dimensional nature of the mass motion). The reason is that when we carry out the transformation of $p(s)$ to $p'(V)$ we have to evaluate $p'(V) = p(s)\,\mathrm{d}s/\mathrm{d}V$, leading to $p'(V) \sim V^{-2/3}V^{-1/3} = V^{-1}$.

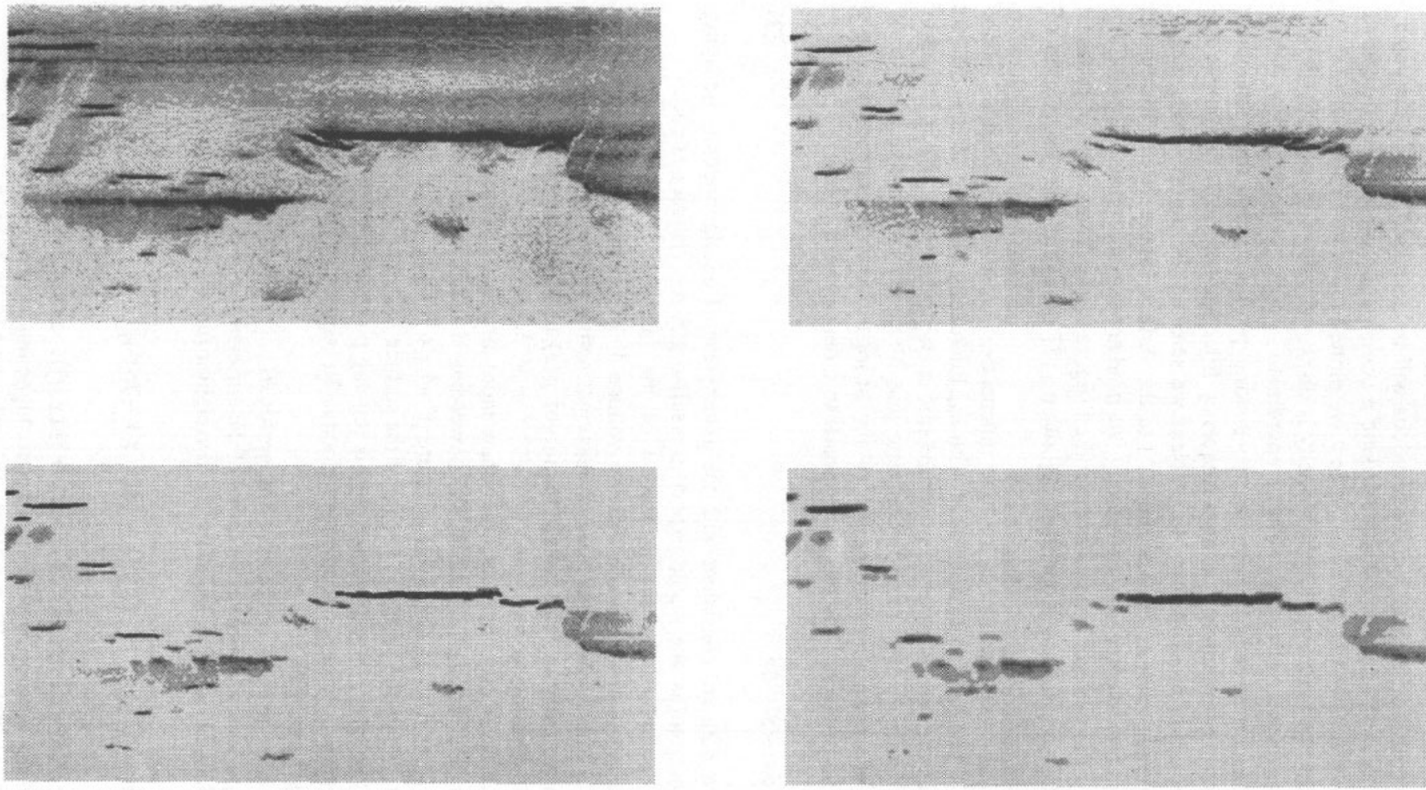
We also used our velocity data sets to obtain more detailed information about the dynamics of the surface roughening due to the power-law distribution of landslides. The distribution of velocities $n(v)$ is displayed in figure 4, where $n(v)\,\mathrm{d}v$ is the number points (pixels) of the profile where the vertical velocity of the surface falls into the interval v, $v+\mathrm{d}v$. The straight part in the large-velocity region in this log–log plot indicates the non-Gaussian nature of the fluctuations of the local surface velocity. Recently similar results have been obtained for wetting fronts [19, 20].

Finally, we made an analysis of the evolving profile $h(x, t)$ (where h denotes the height of the profile at the position x at time t), in terms of the correlations of the height fluctuations in time. This was carried out by calculating the correlation function [2–4]

$$c(\Delta h) = \left\langle \left[\left(h(x,t) - \bar{h}(t)\right) - \left(h(x,t') - \bar{h}(t')\right) \right]^2 \right\rangle_{x,t,t'}^{1/2} \tag{4}$$

where the average position of the profile is $\bar{h}(t) = \langle h(x,t) \rangle_x$, and $\Delta h = \bar{h}(t) - \bar{h}(t')$, which defines the 'time' in the terminology of the kinetic roughening, instead of the experimental time, t. We found (figure 5) that the correlation function $c(\Delta h)$ scales as

$$c(\Delta h) \sim \Delta h^{\beta} \tag{5}$$



... for the given horizontal positions x at time t for the sinking profile of the granular ridge. In the space-time plots the magnitude of the velocity is shown by using a grey scale (darker shades correspond to larger velocities). The x coordinate is directed horizontally, while the vertical direction corresponds to time. *The figures show the steps of the image processing explained in the text.*

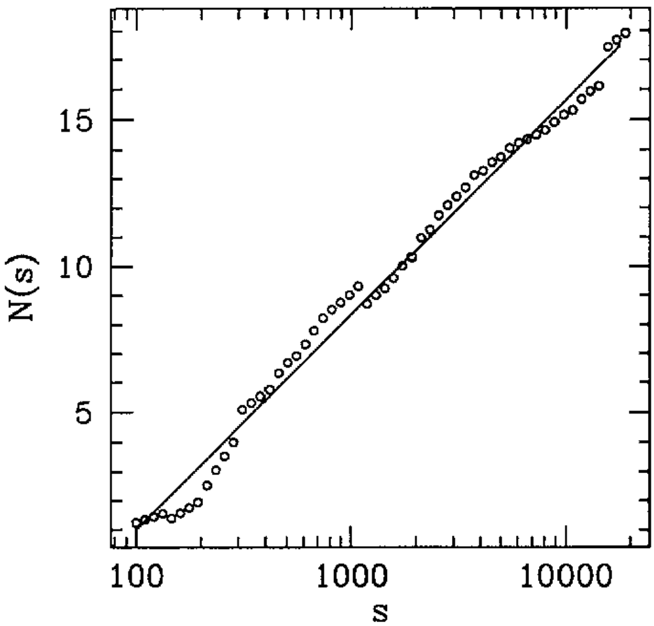


Figure 3. This log–lin plot shows the results of nine independent erosion experiments accumulated into a single distribution of landslide sizes. The data demonstrate that $N(s)$, the cumulative distribution of landslides (clusters of size smaller than s) per experiment, has an almost logarithmic dependence on s in a relatively wide region of sizes.

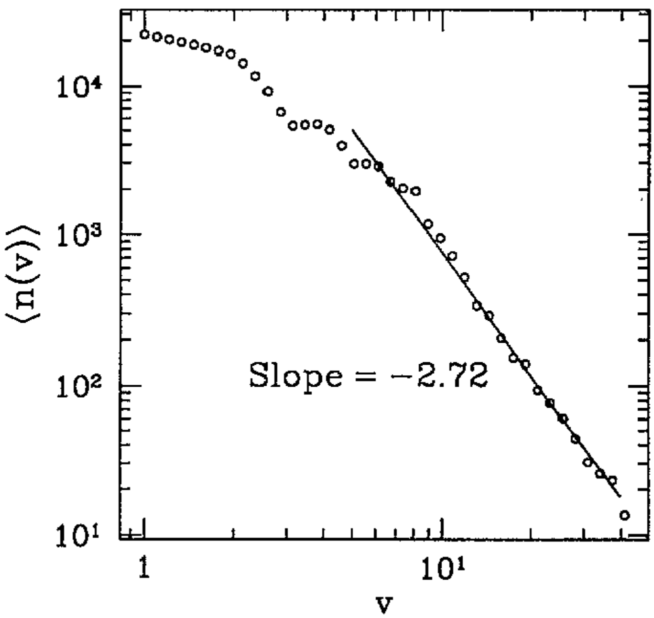


Figure 4. In this figure the distribution of velocities $n(v)$ is displayed, where $n(v)\,dv$ is the number points (pixels) of the profile where the vertical velocity of the surface falls into the interval $[v, v + dv]$. The unit of v is 1 pixel $sec^{-1} \cong 1$ mm sec^{-1}.

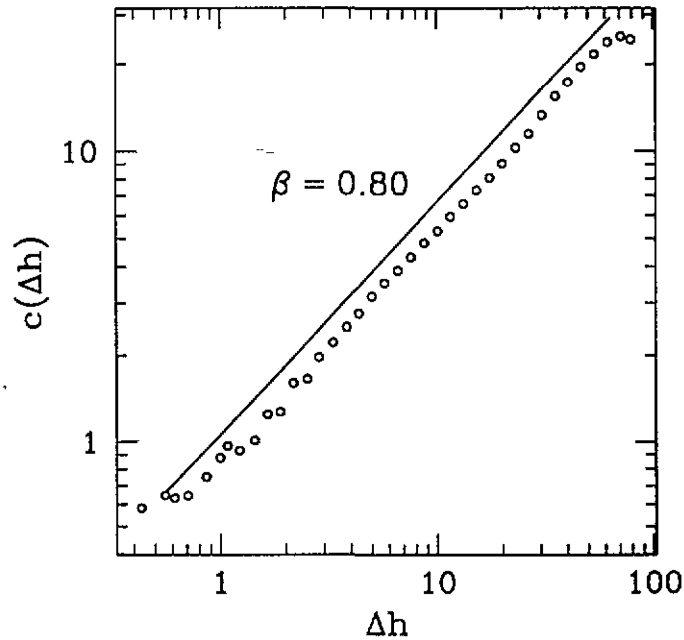


Figure 5. The height correlation function $c(\Delta h)$ (see (4)) has a well defined linear part as a function of the average height difference Δh, indicating that the surface roughens as a function time according to an exponent $\beta \simeq 0.8$.

with an exponent $\beta = 0.8 \pm 0.06$. This value is much more accurate than our preliminary estimate ($\beta \approx 0.9$ [16]).

The results above were stable against the modification of the two experimental parameters: the sand-to-soil ratio and the watering intensity. Our results suggest that the experimental system evolved into a dynamical critical state without fine tuning of the control parameters. This indicates that erosion, self-organized criticality and kinetic roughening are related. The question whether our results are relevant from the point of the size distribution of landslides in actual mountains is intriguing. There are only a few experimental results indicating power-law-like distribution of landslide-type events. Such behaviour was observed during a construction in the Himalayas [21] with various limited scaling regions over which the power-law decay could be associated with exponents ($\tau < -0.3$) smaller than the value we obtained for our model experiment. An algebraic-like size dependence was also detected in the context of rockfalls [22, 23] measured on a cliff in County Antrim, Northern Ireland. In addition, on theoretical grounds one expects universality (size-independent behaviour) in processes with scaling of the spatial and temporal variables describing a complex process. Thus, we expect that our work has relevance to geomorphological processes involving mass motions. In such processes erosion due to precipitation results in rough surfaces and in the studies of the corresponding river network models [24–26] phenomena analogous to self-organized criticality were found.

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